Chapter 17

Optical Fiber Amplifier (EDFA)

17.1 Single-mode optical transmission systems with optical amplifiers

17.1.1 Introductory consideration of the System

According to section RS optical transmission systems are either attenuation or dispersion-limited. At least the loss limitation can be partially canceled out by the introduction of optical amplifiers.
Optical amplifiers use stimulated emission to amplify a optical signal, just as lasers. For example, semiconductor laser amplifier can be constructed easily by using anti-reflection (AR) coated end facets.
Optical fiber amplifiers, however, have acquired a greater importance. A possible transmission system with fiber amplifiers is shown in Fig. 17.1 This enables the possibility of transmitting individual channels at different wavelengths $\lambda_1 \ldots \lambda_N$ (so-called Wavelength-Division Multiplexing). They can be multiplexed (MUX) together in one fiber and then amplified with one optical amplifier (OA) in order to compensate the attenuation. At the receiver the individual channels can be separated by using a demultiplexer (DMUX). In this way, transmission lengths of over 10,000km (Trans-pacific) can be achieved without regenerating the optical signals electrically.

17.1.2 Basic principle of fiber amplifiers

A fiber amplifier consists essentially of a rare earth (e.g. Er) doped fiber (generally quartz glass fiber). This Er-doped fiber is pumped with a laser of an appropriate wavelength (i.e., the laser light is irradiated into the Er-doped fiber). Therefore a population inversion and optical gain can be achieved in the Er-doped fiber.
Figure 17.2 shows the energy level diagram of Er. With a λ = 980 nm pump laser for example, electrons from the ground energy state of $^4I_{1\ 5/2}$ are excited into the energy state $^4I_{1\ 1/2}$. Due to relaxation, the electrons fall (with a time constant of 1 μs) into the energy state $^4I_{1\ 3/2}$.
The time constant at the energy state $^4I_{1\ 3/2}$ is 10 ms and therefore relatively high. This leads to a high population inversion of the electrons even at moderate pump powers. Therefore stimulated amplification is feasible.
In order to achieve an optical amplification, generally pump powers of a few 10 mW are sufficient. Typ-

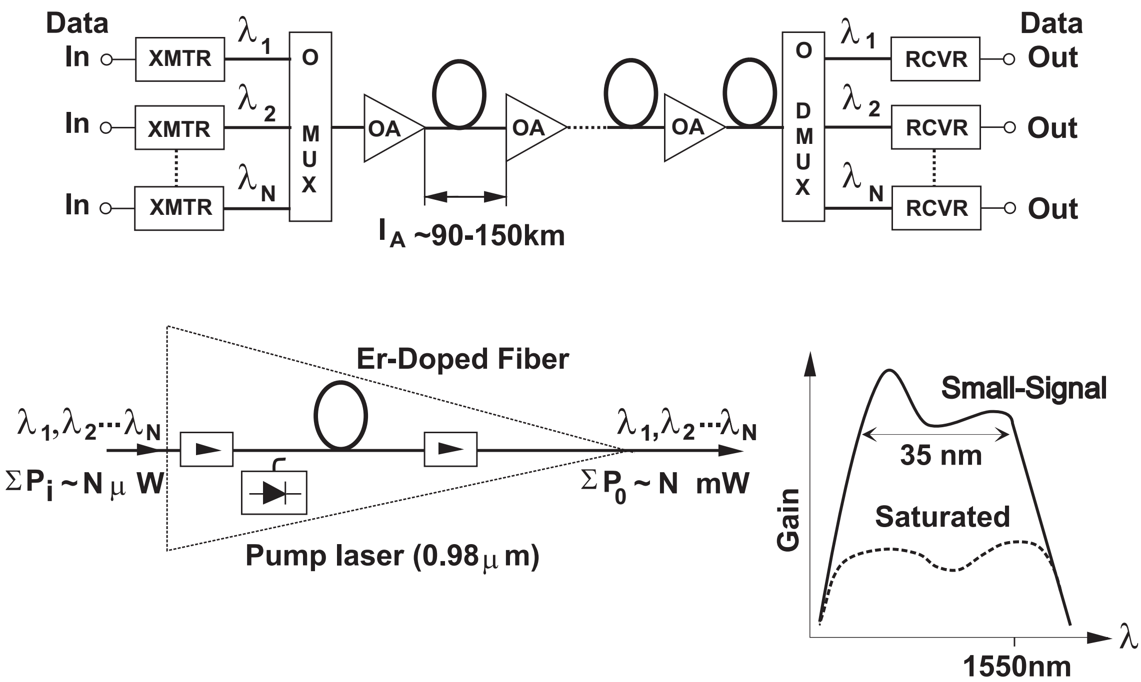

Figure 17.1: Transmission system with optical amplifiers

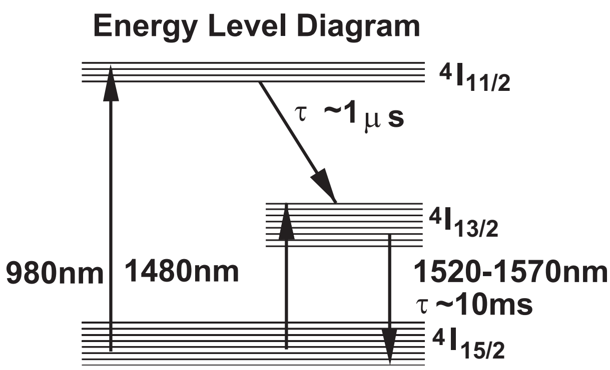

Figure 17.2: Energy level diagram of Er

ically Er-doped fiber amplifiers with the length of approximately 10m can achieve amplification of about $> 20 \ldots 30$ dB. The maximum achievable optical output power of the fiber amplifier is limited by the available pump power, typical Er-doped fiber amplifiers permit saturation output powers of $20 \ldots 50$ mW ($13 \ldots 17$ dBm).
The approximate gain profile of an Er-doped fiber amplifier is shown in Figure 17.1, where a bandwidth of about 35 nm (this corresponds to more than 4 THz = 4000 GHz) is achieved. Compared to electrical amplifiers, fiber amplifiers have thus an extremely high bandwidth.
Er-doped amplifiers are of particular importance, since they allow high amplification already at relatively low pump powers. On the other, they have a flat gain profile at 1550 nm, which is the wavelength at which quartz glass fiber has its minimal attenuation.
Using other rare earths, fiber amplifiers at other wavelengths (e.g. Pr doping for $\lambda = 1.3\,\mu$m) are possible.

17.1.3 Noise by transmission lines with fiber amplifiers

Unfortunately, the fiber amplifiers not only amplify the input optical signal, but they increase, due to the spontaneous emission (which is always associated with stimulated emission), also the noise level and this can degrade the signal-to-noise ratio.
To analyze the noise of the fiber amplifier, initially the quantum noise is considered in detail.

17.1.4 Quantum noise

The quantum noise behaves similar to shot noise (see lecture Hochfrequenztechnik II, chapter RAU). If the total optical power $P(t)$ is represented by

$$P(t) = P_0 + \Delta P(t) \tag{17.1}$$

with the average optical power P_0 and the fluctuation $\Delta P(t)$, due to quantum noise, the one-sided power spectral density (variance per effective noise bandwidth) of $\Delta P(t)$ can be defined similar to the shot noise. Replacing the average current by P_0 and the elementary charge by $(h\nu)$, the PSD results in:

$$\frac{d < (\Delta P)^2 >}{df} = 2 \cdot (h\nu) P_0 \tag{17.2}$$

i.e. with the autocorrelation function

$$\rho_{\Delta P}(\tau) =< \Delta P(t) \Delta P(t-\tau) >= (h\nu) P_0 \delta(\tau) \tag{17.3}$$

The (two-sided) power spectral density (PSD) is obtained as the Fourier transform of the autocorrelation function

$$S_{\Delta P} = \int_{-\infty}^{\infty} < \Delta P(t) \Delta P(t-\tau) > \exp(-j\omega\tau) d\tau = (h\nu) P_0 \ , \tag{17.4}$$

while for the (one-sided) PSD of Eq. (17.2) applies:

$$\frac{d < (\Delta P)^2 >}{df} = 2 \cdot S_{\Delta P} \tag{17.5}$$

17.1.5 Description of quantum noise with the zero-point energy

The description of the quantum noise is directly possible with the complex optical field $E(t)$, which is described as:

$$E(t) = (E_0 + \Delta E_0)\exp(j2\pi\nu_0 t) \tag{17.6}$$

E_0 denotes a monochromatic optical field with the frequency ν_0, that is normalized, so that $|E_0|^2 = P_0$ applies (P_0 - optical power). $\Delta E_0(t)$ denotes the superposed fluctuations due to quantum noise. The fluctuation ΔE_0 can be described by the autocorrelation function

$$< \Delta E_0(t)\Delta E_0^\star(t-\tau) >= \frac{h\cdot\nu}{2}\delta(\tau)\ , \tag{17.7}$$

where $h\cdot\nu/2$ is the so-called ”zero-point energy”, so that the one-sided noise power spectral density (PSD)

$$S_{\Delta E_0} = \int_{-\infty}^{\infty} < \Delta E_0(t)\Delta E_0^\star(t-\tau) > \exp(-j\omega\tau)d\tau = \frac{h\nu}{2} \tag{17.8}$$

equates to the zero-point energy.
ΔE_0 is complex and therefore given as:

$$\Delta E_0 = Re(\Delta E_0) + jIm(\Delta E_0)\,, \tag{17.9}$$

so that the total noise PSD of Eq. (17.8) is divided in equal parts. The real part and the imaginary part:

$$S_{Re(\Delta E_0)} = S_{Im(\Delta E_0)} = \frac{h\cdot\nu}{4}\ . \tag{17.10}$$

The optical power $P(t)$ can be obtained, by using Eq. (17.6), as

$$\begin{aligned} P(t) &= |E(t)|^2 = |E_0|^2 + 2Re(E_0\Delta E_0^\star) + |\Delta E_0|^2 \\ &= P_0 + \Delta P(t)\ , \end{aligned} \tag{17.11}$$

where, assuming $|E_0| \gg |\Delta E_0|$, the term $|\Delta E_0|^2$ in Eq. (17.11) can be neglected. Hence, it results in

$$\Delta P(t) = 2Re(E_0\Delta E_0^\star) \tag{17.12}$$

It it assumed that E_0 is real (for simplicity), $\Delta P(t) = 2E_0 Re(\Delta E_0)$ applies. Thus, the noise PSD is of the form

$$S_{\Delta P} = 4E_0^2 S_{Re(\Delta E_0)} = 4P_0\frac{h\cdot\nu}{4} = (h\cdot\nu)P_0 \tag{17.13}$$

in accordance to Eq. (17.4). Therefore it is proven, that the quantum noise can be described by the zero-point fluctuation ΔE_0.

17.1.6 Noise factor (NF) of fiber amplifiers

The noise behavior of semiconductor laser - or fiber amplifiers results from Fig. 17.3: The input signal E_0 is amplified by the gain G (i.e. amplitude gain $\sqrt{G}$). Additionally, the zero-point fluctuations ΔE_0 witht the autocorrelation function given by Eq. (17.7), appears at the output. These are independent of the amplification process. Furthermore, the gain, provided by the stimulated emission, is always combined with a spontaneous emission, which occurs as additive noise. This is considered in Fig. 17.3 as ΔE_{ASE} (ASE = amplified spontaneous emission). ΔE_{ASE} can be described by the autocorrelation function

$$< \Delta E_{ASE}(t) \Delta E^{\star}_{ASE}(t-\tau) >= n_{sp}(G-1)(h \cdot \nu)\delta(\tau) \tag{17.14}$$

i.e. the (two-sided) power spectral density

$$S_{\Delta E_{ASE}} = n_{sp}(G-1)(h \cdot \nu) \tag{17.15}$$

Eq. (17.14), (17.15) are not derived in this this lecture, but they can be described intuitively, if one considers, that the spontaneous emission rate is proportional to the stimulated emission rate (and therefore to $(G-1)$) and to the inversion coefficient n_{sp} (compare to e.g. S. MOD/1).
Similar to an electric amplifier, a noise factor F can be defined as:

$$F = \frac{(SNR)|_{Eingang}}{(SNR)|_{Ausgang}} = \frac{|E_0|^2 / < |\Delta E_0|^2 >}{G|E_0|^2/(< |\Delta E_0|^2 > + < |\Delta E_{ASE}|^2 >)} \ , \tag{17.16}$$

where SNR describes the signal-to-noise ratio. By using Eq. (17.16), the following equation can obtained:

$$F = \frac{1}{G}\left(1 + \frac{< |\Delta E_{ASE}|^2 >}{< |\Delta E_0|^2 >}\right) = \frac{1}{G}\left(1 + 2n_{sp}(G-1)\right) \ , \tag{17.17}$$

if one considers $< |\Delta E_{ASE}|^2 >\sim S_{\Delta E_{ASE}}$ with Eq. (17.15) and $< |\Delta E_0|^2 >\sim S_{\Delta E_0}$ with Eq. (17.8).
For high gain $G \gg 1$, Eq. (17.17) can be simplified to:

$$F \approx 2 \cdot n_{sp} \ . \tag{17.18}$$

The smallest possible noise factor F of a fiber amplifier (n_{sp}=1) is therefore 2 ($\equiv$ 3 dB).
The noise factor F in Eq. (17.16) is related to the optical signal-to-noise ratio (OSNR). It can be shown, that the noise factor F remains unchanged, if the signal-to-noise ratio are related to the electric power after the opto-electric conversion.

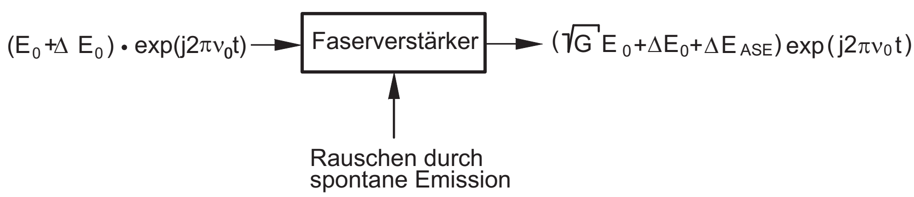

Figure 17.3: Representation of noise in fiber amplifiers

17.1.7 Examples of noise factors

Passive components

For passive components, e.g. lossy fibers, fiber connectors or other coupling points, the gain is $G_F < 1$. On the other hand $\Delta E_{ASE} = 0$ applies, so that the noise factor F, according to Eq. (17.17), is simply given by

$$F = \frac{1}{G_F} \tag{17.19}$$

(compare the noise factors of passive electric networks, see e.g. Hochfrequenztechnik I, chapter RAU)

Series connection of two optical networks

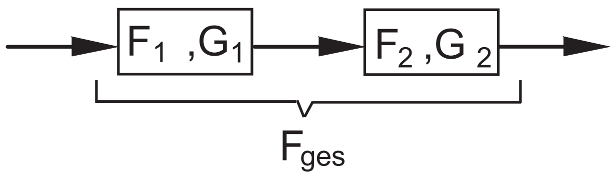

Figure 17.4: Series connection of two optical networks

In analogy to electrical networks (cf. Hochfrequenztechnik I, chapter RAU) it can be shown, that the series connection of two optical networks with the noise factors F_1, F_2 and the gains G_1, G_2 leads, in accordance with Fig. 17.4, to the total noise factor

$$F_{ges} = F_1 + \frac{F_2 - 1}{G_1} \tag{17.20}$$

As an example, we assume a optical amplifier (OA) with the noise factor F_v and a lossy coupling at the input, with the coupling efficiency η and therefore a noise factor $1/\eta$. The total noise factor can thus be obtained as

$$F_{ges} = \frac{1}{\eta} + \frac{F_v - 1}{\eta} = \frac{F_v}{\eta} \tag{17.21}$$

Since the doped fiber of the fiber amplifier and the fiber of the fiber transmission line are generally compatible, high coupling efficiency η close to 1 are feasible. This leads to noise factors of $4 \ldots 5$ dB for fiber amplifiers. In semiconductor laser amplifiers, similar noise factors for the inner amplifier with $n_{sp} \approx 1.5$ (cf. Eq. (17.12)) are achievable. By taking into account the coupling efficiency between the fiber and the active region in the semiconductor laser amplifier, the noise factor increases typically to $8 \ldots 9$ dB.

17.1.8 Discussion of the noise behavior of complete transmission lines

In order to analyze the noise behavior of a complete transmission line, as shown in Fig. 17.1, first the noise factor for one segment is discussed. These segment consists of a fiber element and the subsequent optical amplifier, as shown in Fig. 17.5. The total gain of the segment is 1, since the amplifier compensates the

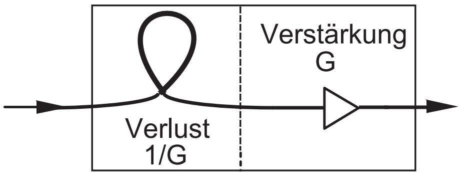

Figure 17.5: Segment of an optical transmission line, consisting of a fiber and the subsequent optical amplifier

losses in the fiber. Due to Eq. (17.21) with $\eta = 1/G$, the noise factor of these segments is thus of the form:

$$F_{seg} = G + G(F_v - 1) = G \cdot F_v \tag{17.22}$$

By using the noise factor F_v of the fiber amplifier, given in Eq. (17.17), following relation is obtained:

$$F_{seg} = 1 + 2n_{sp}(G - 1) \ . \tag{17.23}$$

The complete optical transmission line now consists of many series-connected segments, as shown in Fig. 17.6. Since each element has the gain 1 (the gain and the fiber losses neutralize each other), the total noise

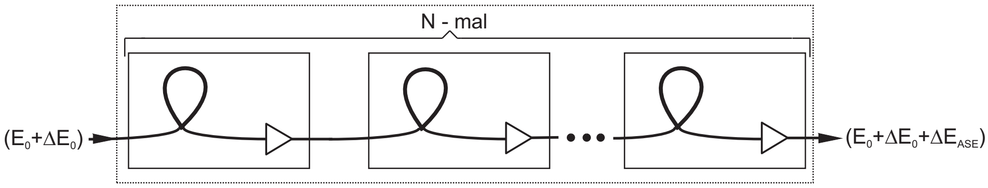

Figure 17.6: Representation of the entire transmission line

factor F_{ges} of the subsequent segments N is given by:

$$F_{ges} = 1 + N \cdot (F_{seg} - 1) = 1 + 2Nn_{sp}(G - 1) \ . \tag{17.24}$$

If L_{oa} denotes the fiber length per segment, the attenuation along the fiber is thus defined as $\exp(-2\alpha L_{oa})$ (Definition of α as on page GRU/8). Therefore the OA has to have the gain of:

$$G = \exp(2\alpha L_{oa}) \tag{17.25}$$

in order to compensate the losses.
Assuming the total transmission length L, $N = L/L_{oa}$ segments are required. The total noise factor for the total transmission length with the amplifier distance L_{oa} is thus of the form:

$$F_{ges} = 1 + 2n_{sp}\frac{L}{L_{oa}}\left(\exp(2\alpha L_{oa}) - 1\right) \ . \tag{17.26}$$

Fig. 17.6 shows the total noise factor for the transmission lengths L=1000 km und L=10000 km as a function of the amplifier distance L_{oa} (Assuming: fiber attenuation $\alpha = 0.2$ dB/km i.e. 0.023 Np/km (GRU/8); $n_{sp} = 2$ corresponding to a noise factor of 6 dB per OA). As shown in Fig. 17.7, the noise figure can increase to very high values for long transmission systems and large OA distances. Values of $F_{ges} > 10000$ are only acceptable in special cases, which are discussed later.

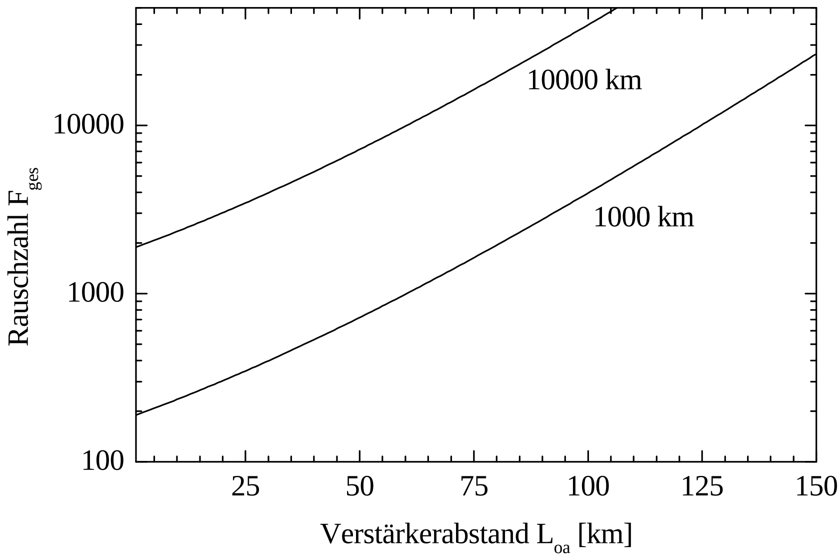

Figure 17.7: Noise factor of an optical transmission line as a function of the amplifier distance

Amplified spontaneous emission at the end of the transmission line

The total transmission line of Fig. 17.6 can be described by the gain $G = 1$ and the noise factor F_{ges}. Therefore the noise PSD (power spectral density) of the amplified spontaneous emission ΔE_{ASE} at the end of the transmission line can be obtained. By using Eq. (17.17), we obtain:

$$< |\Delta E_{ASE}|^2 >= (G \cdot F - 1) < |\Delta E_0|^2 > \tag{17.27}$$

i.e. for the PSD

$$S_{\Delta E_{ASE}} = (G \cdot F - 1) S_{\Delta E_0} \ , \tag{17.28}$$

Assuming a transmission line with $G = 1$ and $F = F_{ges}$, we obtain:

$$S_{\Delta E_{ASE}} = (F_{ges} - 1) S_{\Delta E_0} \ . \tag{17.29}$$

The variance of the ASE $< |\Delta E_{ASE}|^2 >$ is given by

$$< |\Delta E_{ASE}|^2 >= S_{\Delta E_{ASE}} \cdot \Delta\nu \tag{17.30}$$

It is proportional to the optical bandwidth $\Delta\nu$ [1], whereat $< |\Delta E_{ASE}|^2 >$ corresponds exactly to the optical power of the spontaneous emission.
With $S_{\Delta E_0} = h \cdot \nu/2$, according to Eq. (17.8), the optical power of the amplified spontaneous emission at the end of the transmission line results in:

$$< |\Delta E_{ASE}|^2 >= (F_{ges} - 1) \frac{h \cdot \nu}{2} \Delta\nu \ . \tag{17.31}$$

[1]In contrast to the discussion of electrical networks, in this case the two-sided PSD has to be considered, since, in accordance to the definition of ΔE_{ASE}, the positive and negative frequencies of ΔE_{ASE} must be considered separately.

As a numerical value for $\lambda = 1.55$ μm ($\nu = c/\lambda$), the following values can be obtained:

$$< |\Delta E_{ASE}|^2 >= 0.064nW \frac{\Delta\nu}{GHz}(F_{ges} - 1) \; . \tag{17.32}$$

To limit the power of the amplified spontaneous emission (ASE), an optical filter with a very small bandwidth $\Delta\nu$ is used at the end of the transmission line (under certain circumstances optical filters within the optical transmission line are required to avoid saturation of the amplifiers, due to high ASE). For $\Delta\nu = 100$ GHz and $F_{ges} = 1000$, e.g. the ASE-power of $< |\Delta E_{ASE}|^2 >= 6.4$ μW can be obtained.
For the following analysis of the system, we assume, that the signal power is much higher than the ASE-power, thus

$$|E_0|^2 \gg < |\Delta E_{ASE}|^2 > \tag{17.33}$$

applies.
Note: The Equations above (17.27)-(17.32) are valid for one polarization in each case. Due to the detection of both polarizations, the overall result is twice the ASE power.

OSNR

It is customary to characterize the signal quality by the OSNR (optical signal-to-noise-ratio), where the detection of both polarizations is assumed.

$$\text{OSNR} = \frac{G \cdot |E_0|^2}{2 < |\Delta E_{ASE}|^2 >} \tag{17.34}$$

For $F \cdot G \gg 1$ and by using Eq. (17.27), Eq. (17.34) can be simplified to

$$\text{OSNR} = \frac{|E_0|^2}{2F < |\Delta E_0|^2 >} = \frac{|E_0|^2}{F \cdot h \cdot \nu \cdot \Delta\nu} \quad . \tag{17.35}$$

Eq. (17.35) can also by described with $|E_0|^2 = P_0$ as

$$\text{OSNR} = \frac{P_0}{F \cdot h \cdot \nu \cdot \Delta\nu} \quad . \tag{17.36}$$

The OSNR is generally indicated for $\Delta\nu = 12.5$ GHz, leading to the OSNR in dB ($\hat{=} 10\,\text{lg}(\text{OSNR})\text{dB}$) at $\lambda = 1.55\mu m$:

$$\text{OSNR}|_{\text{dB}} = 58_{\text{dB}} + P_0|_{\text{dBm}} - F|_{\text{dB}}. \tag{17.37}$$

For a transmission system, as shown in Fig. 17.6, the noise factor F can be replaced by F_{ges}. Eq. (24) can be simplified with $F_{ges} \gg 1$ to:

$$F_{ges} \simeq N \cdot F_{seg} \tag{17.38}$$

Due to equations (22) and (25), it applies:

$$F_{seg} = \exp(2\alpha L_{oa}) \cdot F_v \tag{17.39}$$

and F_{ges} results in:

$$F_{ges} \simeq N \cdot \exp(2\alpha L_{oa}) \cdot F_v \tag{17.40}$$

The OSNR at the end of the transmission line can thus be described as:

$$\text{OSNR}|_{\text{dB}} = 58|_{\text{dB}} + P_0|_{\text{dBm}} - F_v|_{\text{dB}} - \alpha L_{oa}|_{\text{dB}} - 10\,\text{lg}(N)|_{\text{dB}} \qquad (17.41)$$

Equation (17.41) permits a simple assessment, if, at the end of the transmission line, a sufficiently high OSNR is available.
Example: Assuming an input power of $P_0 = 1\text{mW}(\equiv 0\text{dBm})$, an OA noise factor of $F_v = 6\text{dB}$, a fiber attenuation between the amplifiers αL_{oa} of $20dB$ and $N = 10$ segments, the OSNR can be determined as 22dB.

System analysis of the entire transmission line

The entire transmission line, according to Fig. 17.6, can be discussed as in Fig. 17.3, where the gain is $G = 1$ and ΔE_{ASE} is used appropriate to eqs. (27)-(30).
According to Fig. 17.3, the output power is thus given by

$$P = |E_0 + \Delta E_0 + \Delta E_{ASE}|^2 \; , \qquad (17.42)$$

Assuming a real signal E_0, this leads to

$$P = E_0^2 + 2E_0\left(Re(\Delta E_0) + Re(\Delta E_{ASE})\right) + |\Delta E_0 + \Delta E_{ASE}|^2 \qquad (17.43)$$

with the average power $P_0 = E_0^2$ and the power fluctuation

$$\Delta P = \underbrace{2E_0\left(Re(\Delta E_0) + Re(\Delta E_{ASE})\right)}_{Signal-ASE-Noise} + \underbrace{|\Delta E_0 + \Delta E_{ASE}|^2}_{ASE-ASE-Noise} \qquad (17.44)$$

The fluctuations, due to the amplified spontaneous emission ΔE_{ASE}, are usually much larger than the zero-point fluctuations ΔE_0. The fluctuations of the optical power ΔP in Eq. (17.44) can thus be separated into two components:

1. Signal-ASE-Noise
2. ASE-ASE-Noise

In general, both values have to be taken into account. If small optical filters are used, (small $\Delta\nu$ in Eq. (17.32)) $< |\Delta E_{ASE}|^2 >$ is relatively small and Eq. (17.33) applies. Hence, the ASE-ASE-Noise can be neglected in comparison to the Signal-ASE-Noise. ΔP can thus be approximated by:

$$\Delta P = 2E_0\left(Re(\Delta E_0) + Re(\Delta E_{ASE})\right) \qquad (17.45)$$

For the two-sided noise power spectral density of ΔP one can obtain in analogy to Eq. (17.13):

$$S_{\Delta P} = 2P_0(S_{\Delta E_0} + S_{\Delta E_{ASE}}) \; , \qquad (17.46)$$

by using Eq. (17.29), yields:

$$S_{\Delta P} = 2P_0 \cdot S_{\Delta E_0} \cdot F_{ges} = P_0 \cdot (h \cdot \nu) F_{ges} \; , \qquad (17.47)$$

For the noise factor $F_{ges} = 1$, this results again in the quantum noise, described by Eq. (17.13) i.e. Eq. (17.4). The one-sided noise PSD is thus given by:

$$\frac{d < (\Delta P)^2 >}{df} = 2S_{\Delta P} = 2P_0(h \cdot \nu)F_{ges} \; . \tag{17.48}$$

The optical power P, in an optical receiver with the diode sensitivity

$$E_p = \eta e/(h \cdot \nu) \tag{17.49}$$

(η- quantum efficiency, e-elementary charge), is converted into the corresponding photocurrent

$$I_{ph} = E_p \cdot P \tag{17.50}$$

The noise PSD of the photocurrent (by neglecting the possibly occuring shot noise) can thus be determined as

$$\frac{d < i_R^2 >}{df} = E_p^2 \frac{d(\Delta P)^2}{df} = 2eI_{ph}\eta F_{ges} \; . \tag{17.51}$$

This noise current is generally very high, e.g. assuming $I_{ph} = 1$ mA and $\eta F_{ges} = 1000$ yields the value

$$\sqrt{\frac{d < i_R^2 >}{df}} = 566 \text{ pA}/\sqrt{\text{Hz}}$$

which normally far exceeds the subsequent amplifier (see chapter OE). One can therefore assume, that the signal-to-noise ratio i.e. the bit error rate is governed only by the fluctuations of the optical power, according to Eq. (17.48). If we discuss a binary pulse code modulation with a modulation between the powers $P_0 = 0$ and P_1, the bit error rate, assuming an optimal decision maker, can be described by a parameter Q

$$Q = \frac{P_1}{\sqrt{< \Delta P_0^2 >} + \sqrt{< \Delta P_1^2 >}} \tag{17.52}$$

as discussed in Eq. (OE 57). For the bit error rate of 10^{-9}, a $Q > 6$ is required. Since the fluctuation variance of the optical power is proportional to the mean optical power (similar to the shot noise), for this ideal representation $< \Delta P_0^2 >= 0$ and by using Eq. (17.48), yields:

$$< \Delta P_1^2 >= 2P_1(h \cdot \nu)F_{ges}\frac{B}{2} \; , \tag{17.53}$$

where $B/2$ (B-Bit rate) was chosen as the electrical receiver bandwidth. Therefore, by using the idealized conditions, Eq. (17.52) results in the following Equation:

$$Q = \sqrt{\frac{P_1}{(h \cdot \nu)B \cdot F_{ges}}} \; , \tag{17.54}$$

At the wavelength $\lambda = 1.55$ μm, the following value can be determined:

$$Q \approx 2800\sqrt{\frac{P_1}{mW}}\sqrt{\frac{Gbit/s}{B}}\frac{1}{\sqrt{F_{ges}}} \; . \tag{17.55}$$

Thus, for example, assuming a bit rate $B = 10$ Gbit/s, $F_{ges} = 4000$ and $P_1 = 1$ mW, results in a $Q = 14$, which represents a high-quality transmission.
Eq. (17.52) applies for the optimum decision threshold, that should be at $P_0 = 0$ because of $< \Delta P_0^2 >= 0$. Unfortunately typical receivers have rather a decision threshold in the middle between P_0 and P_1, leading to a parameter Q with a half of this value above.

The achievable transmission quality depends, due to Eq. (17.54),(17.55), also significantly on the optical power P_1 at the output of each amplifier within the transmission line. This power is upwardly limited by the achievable amplifier output power as well as non-linear effects in the transmission fiber, e.g. Brillouin scattering, Raman scattering as well as cross-phase modulation (XPM), self-phase modulation (SPM) and four-wave mixing due to the Kerr-effect (also called quadratic electro-optic effect (QEO)).
But even at moderate optical powers large transmission distances are possible. In the above-mentioned example, the noise factor of $F_{ges} = 4000$, due to Fig. (17.7), corresponds, with an OA distance of $L_{oa} = 30$ km, to a total transmission distance of 10.000 km (Trans-Pacific).
The required OSNR can now be specified to avoid exceeding the bit error rate i.e. falling below a given Q.

Required OSNR

In order to obtain a specific Equation for OSNR, Eq. (17.54) can be expanded with $\Delta\nu$:

$$Q = \sqrt{\frac{P_1}{(h \cdot \nu)\Delta\nu \cdot F_{ges}} \cdot \frac{\Delta\nu}{B}}. \tag{17.56}$$

For the binary OOK modulation (OOK: on-off-keying, i.e. the switching on and off of the optical power), assuming the average power $P_0 = P_1/2$, Eq. (17.56) results in:

$$Q = \sqrt{\frac{2P_0}{(h \cdot \nu)\Delta\nu \cdot F_{ges}} \cdot \frac{\Delta\nu}{B}} = \sqrt{2\,\mathrm{OSNR}\frac{\Delta\nu}{B}}, \tag{17.57}$$

where Eq. (17.36) has been used. As a result of Eq. (17.57), a relation between the error rate (expressed by Q) and the required OSNR can be obtained.

Example: For a binary OOK with $B = 42.7Gbit/s$ and a bit error rate $BER = 10^{-3}$ ($Q = 3.1$), a required OSNR (with $\Delta\nu = 12.5GHz$) of 16.4 ($\equiv 12.2dB$) can be determined by using Eq. (17.57). In fact, this estimated OSNR value is rather too optimistic, since the included approximation, made above, are significant, e.g. the neglecting of the ASE-ASE noise.
By using more accurate numerical calculations, the OSNR values for 42.7 Gb/s and BER $= 10^{-3}$, indicated in the following table, can be determined:
OOK and DPSK are binary modulation types, while DQPSK is a quaternary modulation type (cf. HFT II / MOD).

Modulation type	OSNR (42.7Gb/s,BER=10^{-3})
NRZ-OOK	15.9 dB
50% RZ-OOK	14.4 dB
50% RZ-DPSK	11.1 dB
50% RZ-DQPSK	12.2 dB

Table 17.1: Required "Optical Signal/Noise Ratio" (referred to $\Delta\nu = 12.5$ GHz) at the data rate of 42.7 Gb/s. (N)RZ: (Non) return-to-zero; OOK: On/off-keying; DPSK: differential phase shift keying; DQPSK: differential quadrature PSK. (P.J. Winzer, R.J. Essiambre: "Advanced Optical Modulation Formats", Proceedings IEEE, vol. 94, May 2006, pp. 952 - 985).